# Rich Vehicle Routing Problem: Survey

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The Vehicle Routing Problem (VRP) is a well known research line in the optimisation research community. Its different basic variants have been widely explored in the literature. Even though it has been studied for years, the research around it is still very active. The new tendency is mainly focused on applying this study cases to real life problems. Due to this trend, the Rich VRP arises; combining multiple constraints for tackling realistic problems. Nowadays, some studies have considered specific combinations of real-life constraints to define the emerging Rich VRP scopes. This work surveys the state-of-the-art in the field, summarising problem combinations, constraints defined and approaches found.

Categories and Subject Descriptors: A.1 [**Introductory and Survey**]; G.2.1 [**Discrete Mathematics**]: Combinatorics—*Combinatorial algorithms*; G.2.3 [**Discrete Mathematics**]: Applications

General Terms: Algorithms, Economics

Additional Key Words and Phrases: Routing, Transportation, Vehicle Routing Problem

## 1. INTRODUCTION

Road transportation is the predominant way of transporting goods in Europe and in other parts of the world. Direct costs associated with this type of transportation have increased significantly since 2000, and more so in recent years due to rising oil prices. Furthermore, road transportation is intrinsically associated with a good deal of indirect or external costs, which are usually easily observable congestion, pollution, security- and safety-related costs, mobility, delay time costs, etc. However, these costs are usually left unaccounted because of the difficulty of quantifying them [Sinha and Labi 2011]. For example, traffic jams in metropolitan areas constitute a serious challenge for the competitiveness of industry: for instance, according to some studies [EC 2008, 2011; Van Essen et al. 2011], external costs due to traffic jams could repre-

sent about 1–2% of the European GDP, a percentage which continues to increase. In addition to these easily observable costs, many others might be considered. In this scenario, it becomes evident that new methods must be developed to support the decision-making process so that optimal (or quasi-optimal) strategies can be chosen in road transportation. This need for optimising the road transportation affects both the public and private sectors, and constitutes a major challenge for most industrialised regions.

Recent advances on Information and Communications Technologies (ICT) —such as the growing use of GPS and smart-phone devices, Internet-scale (distributed) systems, and Internet computing technologies—, open new possibilities for optimising the planning process of road transportation [Orozco 2011]. In particular, when combined with advanced Simulation and Optimisation techniques, Distributed- and Parallel-Computing Systems (DPCS) allow the practical development and implementation of new ICT-based solutions to support decision-making in the Transportation and Logistics (T&L) arena. "Real-world applications, both in North America and in Europe, have widely shown that the use of computerised procedures generates substantial savings (generally from 5% to 20%) in the global transportation costs" [Toth and Vigo 2002]. Road-transportation optimisation (cost-saving) issues are especially critical in the case of Small and Medium Enterprises (SME), since they are rarely able to obtain the economic and human resources required to implement, maintain, and manage efficient routing-optimisation methods. Similarly, those companies have difficulties to access the appropriate technologies —e.g., computer clusters and expensive commercial software—, which would help them to improve their productivity level and to reduce the unnecessary costs, thus making a more sustainable business model.

### 1.1. Context and Motivation

In this context, the goal of the so-called Vehicle Routing Problem (VRP) is to optimise the routing design (distribution process from depots to customers) in such a way that customers' demand of goods is satisfied without violating any problem-specific constraint —e.g., route maximum distance or time-related restrictions [Golden et al. 2008]. The VRP has many variants depending on the parameters and constraints considered. Despite its apparent simplicity, in computational complexity theory, the classical version of VRP and its variants (for extension) are NP-hard (non-deterministic polynomial-time hard) [Lenstra and Rinnooy-Kan 1981]. This implies that, in practice, it will not be possible to guarantee the (mathematically) optimal solution. Meaning that the given problem cannot be solved by an algorithm in a finite number of steps [Garey and Johnson 1979]. NP-hard problems may be of any type: decision problems, search problems, or optimisation problems. Some practical examples could be found in Data mining, Scheduling, Planning, Decision support, etc. In recent years, due to the fast development of new and more efficient optimisation and computing methods, the interest of academics and practitioners has been shifting towards realistic VRP variants, which are commonly known as Rich VRP. These problems deal with realistic (and sometimes multi-objective) optimisation functions, uncertainty (i.e., stochastic or fuzzy behaviours), dynamism, along with a wide variety of real-life constraints related to time and distance factors, use of heterogeneous fleets, linkage with inventory and scheduling problems, integration with ICT, environmental and energy issues, etc.

After a number of VRP variants have appeared over the years, we have found a need to classify those which can be part of the Rich VRP. As a matter of fact, there is no consensus on which problems can be described as Rich ones and which are just a new variant of VRP. Thus, in this paper we describe the main variants of the VRP, analyse their constraints and present the main techniques used to face them. This work allows us to create an extensive list of the main constraints that are applicable to a Rich VRP problem. Furthermore, we try to introduce a definition of the Rich VRP summarising

all the previous information existing about the problem. Furthermore, a classification of the Rich VRP problems and a matrix table relating all the Rich VRP papers with the collected constraints are shown. Finally, there is summed up the future trends of both vehicle routing problems and tools used to solve them.

### 1.2. Structure

To begin with, a definition of the basic problem, the Capacitated Vehicle Routing Problem is given in Section 2. Additionally, we introduce the basic formulation of the problem and the main classic variations of the VRP. In Section 3 the most common approaches for solving the current variations of the VRP are introduced. In Section 4 a definition of the Rich Vehicle Routing Problem is given. A complete literature review is presented in Section 5. Section 6 presents a complete classification of all the papers on the Rich VRP and the explanation of all the different kinds of constraints which can be found in them. In Section 7, some perspective on current and future trends regarding the Rich VRP is provided. Finally, we summarise the survey together with some conclusions in Section 8.

### 2. PROBLEM DEFINITION

In the Capacitated Vehicle Routing Problem (CVRP), first defined by [Dantzig and Ramser 1959], a homogeneous fleet of vehicles supplies customers using resources available from a depot or central node (see Fig. 1). Each vehicle has the same capacity (homogeneous fleet) and each customer has a certain demand that must be satisfied. Additionally, there is a cost matrix that measures the costs associated with moving a vehicle from one node to another. These costs usually represent distances, travelling times, number of vehicles employed or a combination of these factors.

   More formally, we assume a set $\Omega$ of $n+1$ nodes, each of them representing a vehicle destination (depot node) or a visit (demanding node). The nodes are numbered from $0$ to $n$, node $0$ being the depot and the remaining $n$ nodes the visits to be performed ($\Omega* = \Omega - \{0\}$). A demand $q_i > 0$ of some commodity has been assigned to each non-depot node $i$, $i \in \Omega*$ (we assume $q_0 = 0$). On the other hand, $A = \{(i,j) : i,j \in \Omega; i < j\}$ represents the set of the $n \cdot (n+1)/2$ existing edges connecting the $n+1$ nodes. Each of these links has an associated aprioristic cost, $c_{ij} > 0$, which represents the cost of sending a vehicle from node $i$ to node $j$. In this original version, these $c_{ij}$ are assumed to be symmetric ($c_{ij} = c_{ji}, 0 \leq i,j \leq n$), and they are frequently expressed in terms of the Euclidean distance ($d_{ij}$) between the two nodes. The delivery process is to be carried out by a fleet of $V$ vehicles ($V \geq 1$) with equal capacity, $Q >> \max\{q_i : i \in \Omega\}$. These $V$ vehicles are responsible of $R$ routes ($R \leq V$).

   Some additional constraints associated to the CVRP are the following [Laporte et al. 2000]:

— Each non-depot node is supplied by a single vehicle.
— All vehicles begin and end their routes at the depot (node 0).
— A vehicle cannot stop twice at the same non-depot node.
— No vehicle can be loaded exceeding its maximum capacity.

   The following generic formulation is based on the formulation proposed by [Toth and Vigo 2002] and then used in [Baldacci et al. 2008] for the heterogeneous fleet VRP variant. It is useful for both symmetrical and asymmetrical instances, as well as for both homogeneous and heterogeneous fleet. The vehicle fleet $M$ is composed by $m$ different vehicle types ($M = \{1, \ldots, m\}$). Each type $k \in M$ has $m_k$ available vehicles at the depot, each having a capacity defined by $Q_k$. There is a three-index binary variable for each edge and possible vehicle type (Eq. 8). The variable $x_{ij}^k$ indicates if the arc $(i,j)$

$(i, j \in \Omega)$ is used or travelled by a vehicle of type $k$ in the optimal solution. In addition, flow variables $y_{ij}^k$ represent the load in the vehicle servicing customer $j$ after visiting customer $i$.

$$\min \sum_{k \in M} \sum_{\substack{i,j \in \Omega \\ i \neq j}} c_{ij}^k x_{ij}^k \tag{1}$$

subject to:

$$\sum_{j \in \Omega*} x_{0j}^k = \sum_{i \in \Omega*} x_{i0}^k \qquad \forall k \in M \tag{2}$$

$$\sum_{k \in M} \sum_{i \in \Omega} x_{ij}^k = 1 \qquad \forall j \in \Omega* \tag{3}$$

$$\sum_{i \in \Omega} x_{iu}^k = \sum_{j \in \Omega} x_{uj}^k \qquad \forall u \in \Omega*, \forall k \in M \tag{4}$$

$$\sum_{j \in \Omega*} x_{0j}^k \leq m_k \qquad \forall k \in M \tag{5}$$

$$\sum_{i \in \Omega} y_{ij}^k - \sum_{i \in \Omega} y_{ji}^k = q_j \sum_{i \in \Omega} x_{ij}^k \qquad \forall j \in \Omega*, \forall k \in M \tag{6}$$

$$0 \leq q_j x_{ij}^k \leq y_{ij}^k \leq (Q_k - q_i) x_{ij}^k \quad \forall i,j \in \Omega, i \neq j, \forall k \in M \tag{7}$$

$$x_{ij}^k \in \{0, 1\} \qquad \forall i,j \in \Omega, i \neq j, \forall k \in M \tag{8}$$

The objective function in Eq. 1 minimises the total distance cost required to service all customers. Eq. 2 implies that the number of vehicles leaving the depot is the same as the number of vehicles returning to it. Eq.'s 3 and 4 ensure that each customer is visited exactly once, and that if a vehicle visits a customer it must also depart from it. Eq. 5 imposes that the number of used vehicles does not exceed the number of available vehicles for each vehicle type. Eq. 6 states that the quantity of goods in the vehicle arriving at customer $j$, $y_{ij}^k$, minus the demand of that customer, equals the quantity of goods in the vehicle leaving it after the service has been completed. Eq. 7 guarantees lower and upper bounds ensuring that: the quantity of goods in the vehicle leaving customer $i$, $y_{ij}^k$, is equal to or greater than the demand of its next visit, $q_j$; and the total demand serviced by each vehicle of type $k$ does not exceed its capacity $Q_k$.

### 2.1. Vehicle Routing Problem Variants

Different variants of the VRP have been target studies in the last fifty years [Laporte 2009]. In the literature, the variants of the VRP include a large family of specific optimisation problems. For instance, the VRP with Time Windows (VRPTW) is one of the most popular families studied in the community [Bräysy and Gendreau 2005a,b]. As main common feature, they are focused in considering one or few constraints into their mathematical models; this has created a huge set of separated branches of VRP research lines with long abbreviation names. Each research line has been identified by the acronym of the considered constraints or attributes inside the optimisation problem. Many of these individual branches have been recombined creating new 'basic' branches. The main variants of the VRP can be found in [Toth and Vigo 2002; Golden et al. 2008]. A relatively new variant that is not included in the aforementioned references is the Green VRP [Erdoğan and Miller-Hooks 2012; Kopfer et al. 2014]. So far the most common current extensions studied in the literature are described here:
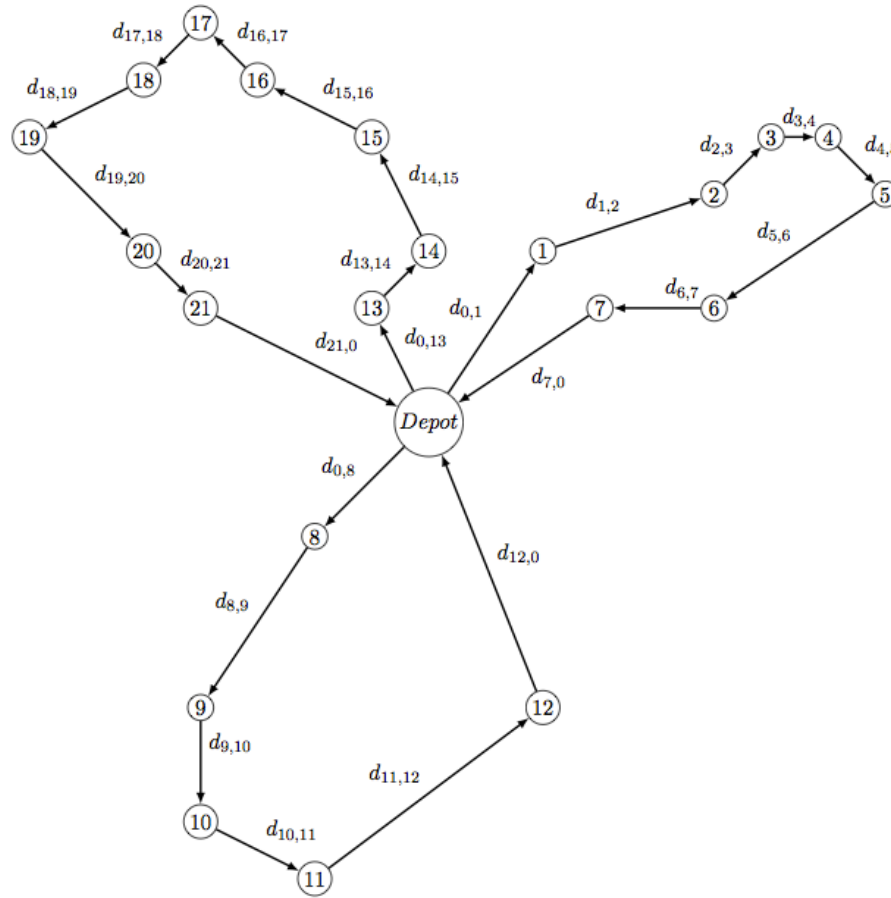
Fig. 1.   Representation of a VRP example using a node design

*Asymmetric cost matrix VRP (AVRP).* The cost for going from customer $i$ to $j$ is different than for going from $j$ to $i$.

*Distance-Constrained VRP (DCVRP).* The total length of the arcs in a route cannot exceed a maximum route length. This constraint can either replace the capacity constraint or supplement it.

*Heterogeneous fleet VRP (HVRP).* The company uses different kinds of vehicles and the routes have to be designed according to the capacity of each vehicle. Some costs could be considered and the number of vehicles could be limited or not, creating different contexts. When the number of vehicles is unlimited then it is called Fleet Size and Mix VRP (FSMVRP). If a specific type of vehicle cannot reach some clients for any reason then the problem becomes a Site-Dependent VRP (SVRP). Also if a vehicle is allowed to perform more than one trip then we are solving a HVRP with Multiple use of vehicles (HVRPM).

*Multiple Depots VRP (MDVRP).* A company has several depots from which they can serve their customers. Therefore, some routes will have different starting/ending

points.

*Open VRP (OVRP).* The planned routes can end on several points distinct to the depot location.

*Periodic delivery VRP (PVRP).* The optimisation is done over a set of days (while normally is daily planned). The customers may not have to be visited each day. Customers can have different delivery frequencies.

*Pickup-and-delivery VRP (PDVRP).* Each customer is associated by two quantities, representing one demand to be delivered at the customer and another demand to be picked up and returned to the depot. In addition to the constraint that the total pickup and total delivery on a route cannot exceed the vehicle capacity, also it has to ensure that this capacity is not exceeded at any point of the route. One variant of the pickup and delivery problem is when the pickup demand is not returned to the depot, but should be delivered to another customer —e.g., transport of people. In some cases, the vehicles must pickup and deliver items to the same customers in one visit (Simultaneous Pickup-and-delivery VRP) —i.e., new and returned bottles. Notice that other important variant is the $1 - M - 1$ ("one-to-many-to-one"), this means that all delivery demands are initially located at the depot, and all pickup demands are destined to the depot. Taken collectively, all delivery demands can be viewed as a single commodity, and all pickup demands can be viewed as a second commodity. This variant is generally referred in the literature as $Delivery - and - Collection$ [Gribkovskaia and Laporte 2008].

*Split-delivery VRP (SDVRP).* The same customer can be served by different vehicles if it will reduce the overall cost. This relaxation of the basic problem is important in the cases where a customer order can be as large as the capacity of the vehicle.

*Stochastic VRP.* There is a realistic aspect of the routing problem where a random behaviour is considered. This is typically the presence of a customer, its demand, its service time or the travel time between customers. So far, this uncertainty aspect has shown to be a key aspect for future demanding developments [Juan et al. 2011].

*VRP with Backhauls (VRPB).* As in the PDVRP, the customers are divided into two subsets. The first subset contains the linehaul customers, which are customers requiring a given quantity of product to be delivered. The second subset contains the backhaul customers, where a given quantity of inbound product must be picked up. Then all linehaul customers have to be visited before the backhaul customers in a route.

*VRP with Time Windows (VRPTW).* Each customer is associated with a time interval and can only be served within this interval. In this problem the dimension of time is introduced and one has to consider the travel time and service time at the customers. A set of time windows for each customer could be also considered (VRP with Multiple Time-Windows). Also these time windows could be flexible depending on some extra costs (VRP with Soft Time-Windows).

*Green VRP (GVRP).* This variant of the VRP aims at including different environmental issues in the optimisation process, e.g. greenhouse gas emissions [Ubeda et al. 2011], pollution, waste and noise [Bektaş and Laporte 2011], effects of using

'greener' fleet configurations [Juan et al. 2014a], etc. An excellent and updated survey on GRVP can be found in [Demir et al. 2014].

Several hybrid variants have been created in the literature from these basic variants inspired in real-life scenarios. A large number of VRP acronyms have been developed to refer to these combinations of routing restrictions. However, all these new combinations can be encompassed in the larger family of Rich VRP, as we will explain later (Section 4).

## 3. VRP METHODOLOGIES

Different approaches to VRPs have been explored. These range from the use of pure optimisation methods, such as mathematical programming, for solving small- to medium-size problems (about up to 75–100 customers) with relatively simple constraints, to the use of heuristics and metaheuristics that provide near-optimal solutions for medium and large-size problems with more complex constraints. Metaheuristics serve three main purposes: solving problems faster, solving larger problems, and obtaining more robust algorithms. They are a branch of optimisation in Computer Science and Applied Mathematics that are related to algorithms and computational complexity theory. Metaheuristics provide *acceptable* solutions in a reasonable time for solving hard and complex problems [Talbi 2009]. Even though the VRP has been studied for decades and a large set of efficient optimisation methods, heuristics and metaheuristics have been developed [Golden et al. 2008; Laporte 2007], more realistic or Rich VRP problems —such as the VRP with Stochastic Demands or the Inventory VRP— are still in their infancy. Following the proposed division of [Talbi 2009], this large family could be preliminary summarised in a balanced tree presented in Fig. 2.

## 3.1. Exact Methods

From [Talbi 2009], "Exact methods obtain optimal solutions and guarantee their optimality". This type of technique is often applied to small-size instances. This family includes a broad set of methods. There are methods like the family of Branch-and-$X$ (where the $X$ represent the different variants) used for solving Integer Linear Programming (ILP) and Mixed Integer Linear Programming problems (MILP); and also Dynamic Programming which focus on solving complex problems by breaking them down into simpler subproblems [Kok et al. 2010]. Likely, Column Generation is a popular technique used for solving larger linear programming problems, which consists in splitting the given problem into two problems: the master problem and the subproblem [Desaulniers et al. 2005]. This allows to simplify the original problem with only a subset of variables in the master problem. A new variable is created in the subproblem, which will be minimised in the objective function with respect to the current dual variables and the constraints naturally associated to the new variable. The Set Partitioning modelling is other binary variable formulation for each feasible route. This technique is quite general and can consider several constraints at a time [Subramanian et al. 2012; Subramanian 2012]. Constraint Programming (CP) is a programming paradigm that uses constraints to define relations among variables [Van Hentenryck 1989]. It differs from other programming languages, as it is not necessary to specify a sequence of steps to execute to solve a problem, but rather its properties. Models in CP are based in three elements: variables, their corresponding domains, and constraints relating all the variables. The main mechanism for solving a problem using CP is called *constraint propagation*. It works by reducing variables domains, strengthening constraints or generating new ones. This leads to a reduction of search space, making the problem easier to solve by means of search algorithms. [Guimarans et al.

Branch and Bound

Branch and X

Branch and Price

Branch and Cut

Constraint
Programming

Exact
Methods

Dynamic
Programming

A*, IDA*

Optimisation
Methods

Approximation
Algorithms

Approximate
Methods

Metaheuristics

Heuristic
Algorithms

Problem - specific
heuristics
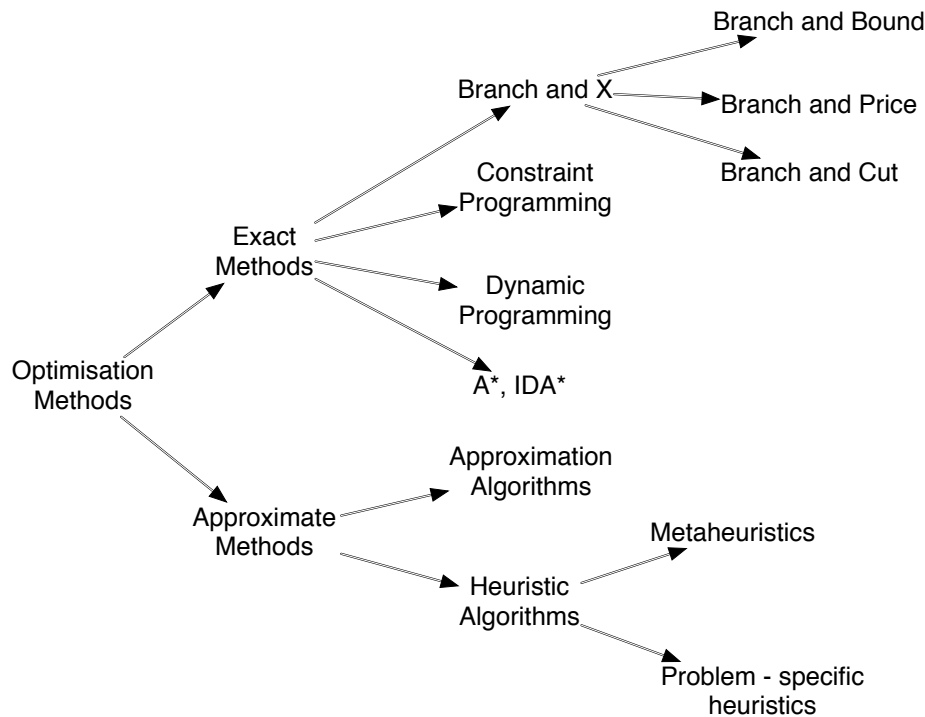
Fig. 2.   Classification of the Classical Optimisation Methods.

2011] presents a hybrid approach to solve the CVRP by applying Lagrange Relaxation
on each route and feasibility checking using a CP model. A* [Hart et al. 1968] is a
computer algorithm commonly used in shortest paths and graph traversal problems,
it uses the best-first search [Dechter and Pearl 1985] to find the most promising node
to expand. In the same way, IDA* [Korf 1985] is a variant of the A* algorithm, that
uses less memory, as it does not keep track of the prior visits, it uses the iterative
deepening. Some of this type of methods are quite popular for the basic CVRP: branch-
and-bound, branch-and-cut, branch-and-price, branch-and-cut-and-price, set partition-
ing based, and dynamic programming. More details of these methods are reviewed in
[Baldacci et al. 2010, 2012; Laporte et al. 2013] for the CVRP and for some of its vari-
ants.

### 3.2. Approximate Methods

From [Talbi 2009], "Heuristics find *good* solutions on large-size problem instances.
They allow to obtain acceptable performance at acceptable costs in a wide range of
problems. They do not have an approximation guarantee on the obtained solutions.
They are tailored and designed to solve a specific problem or/and instance. Meta-
heuristics are general-purpose algorithms that can be applied to solve almost any op-
timisation problem. They may be viewed as upper level general methodologies that
can be used as a guiding strategy in designing underlying heuristics". The author
also proposes that two contradictory criteria must be taken into account: exploration
of the search space (*diversification*) and the exploitation of the best solutions found
(*intensification*). Promising regions are determined by the obtained *good* solutions. In

the intensification, the promising regions are explored more thoroughly in the hope to find better solutions. In diversification, non-explored regions must be visited to be sure that all regions of the search space are evenly explored and that the search is not confined to only a reduced number of regions.

There are many metaheuristics inspired in natural processes like Evolutionary Algorithms (including Genetic Algorithms, GA) and Ant Colony Optimisation (ACO). For instance the ACO metaheuristic is inspired on the communication and cooperation mechanisms among real ants that allow them to find the shortest paths from their nest to food sources. The communication medium is a chemical compound (pheromone). The amount of pheromone is represented by a weight in the algorithm [Gendreau et al. 2008]. In ACO algorithms, the range $[min, MAX]$ of pheromone trail values can be controlled. This type of technique can also be classified as population-based metaheuristic because they iteratively improve a population of solutions. Other member of this wide group is the deterministic strategy of Scatter Search which recombines selected solutions from a known set to create new ones [Talbi 2009].

Other techniques are based on memory usage (short-, medium-, and long-term). Tabu Search (TS) is a local search-based metaheuristic where, at each iteration, the best solution in the neighbourhood of the current solution is selected as the new current solution, even if it leads to an increase in solution cost. A short-term memory (Tabu list) stores recently visited solutions (or attributes) to avoid short-term cycling [Gendreau et al. 2008]. This family can be considered as single-solution based metaheuristic since it is focused on improving a single solution at a time. A common feature is that all include the definition of building an initial solution. Other promising techniques are Variable Neighbourhood Search (VNS) and Greedy Randomised Adaptive Search Procedure (GRASP). VNS has been widely used in several problems. It is based on a successive exploration of a set of predefined neighbourhoods to find a better solution at each step. Large Neighbourhood Search (LNS) [Guimarans 2012] can be interpreted as a special case of VNS where efficient procedures are designed to consider a high number of neighbourhoods at the same time. Inside of this branch, we can find one of the first techniques used for the Travelling Salesman Problem which is the Nearest Neighbourhood. Simulated Annealing (SA) [Nikolaev and Jacobson 2010] is another single-solution based method, which is based on the same physical principle used in the process of heating and then slowly cooling of a substance in order to produce a strong crystalline structure. So it is typical to include a temperature parameter in order to control the process.

There are some approximate algorithms called Heuristics, made with a tailored design to solve a specific problem. Following a systematically number of steps, they are used to find an acceptable solution. However, they do not guarantee to find the optimal solution. For instance, [Clarke and Wright 1964] Savings (CWS) is probably one of the most cited heuristic to solve the CVRP. In the literature, there are several variants and improvements of the CWS [Golden et al. 1984]. The original version of CWS is based on the estimation of possible savings originated from merging routes, i.e., for unidirectional or symmetric edges $Sav(i, j) = c_{i0} + c_{0j} - c_{ij}$. These savings are estimated between all nodes, and then decreasingly sorted. Then the bigger saving is always taken, and used to merge the two associated routes. As the authors propose, this procedure uses the concept of savings. In general, at each step of the solution construction process, the edge with the most savings is selected if and only if the two corresponding routes can feasibly be merged using the selected edge. The CWS algorithm usually provides relatively good solutions in less than a second, especially for small and medium-size problems. In addition, new algorithms have been proposed based on CWS. For instance, [Juan et al. 2010] propose a multi-start randomised approach, called Simulation in

Routing via the Generalised Clarke and Wright Savings heuristic (SR-GCWS), that could be considered a metaheuristic in this general classification.

## 4. RICH VRP DEFINITION

A first attempt to define the Rich VRP (RVRP) was made by [Toth and Vigo 2002]. The authors define the potential of extending the "vehicle flow formulations, particularly the more flexible three-index ones". The authors stated that models of the symmetric and asymmetric CVRP "may be adapted to model some variants of the basic versions". Other authors have given a different adjective to this realistic problem. For the research community, the RVRP is a generalisation or union of other independent problems. As [Goel and Gruhn 2005, 2006, 2008] deal with the General Vehicle Routing Problem (GVRP), "a combined load acceptance and routing problem which generalises the well known Vehicle Routing Problem and Pickup and Delivery Problem. Furthermore, it amalgamates some extensions of the classical models which, up to now, have only been treated independently". On a Special Issue explicitly specialised for Rich models, the editors [Hasle et al. 2006] summarise "non-idealised models that represent the application at hand in an adequate way by including all important optimisation criteria, constraints, and preferences". In fact, [Hasle and Kloster 2007] refers to this type of problem as an *Industrial* or *Applied* Routing Problem.

[Pellegrini et al. 2007] state that "in recent years, moreover, thanks to the increasing efficiency of these methods and the availability of a larger computing power, the interest has been shifted to other variants identified as Rich VRP. The problems grouped under this denomination have in common the characteristics of including additional constraints, aiming a closer representation of real cases". Their case study, is characterised by many different types of constraints, each of which unanimously classified as challenging even when considered alone". For instance, [Crainic et al. 2009b,a] introduce another term to refer RVRP. They deal with the multi-attribute VRP like *rich* problems. They also stated that "Real-world problems are generally characterised by several interacting *attributes*, which describe their feasibility and optimality structures. Many problems also display a combinatorial nature and are, in most cases of interest, both formally difficult and dimensionally large. In the past, the general approach when tackling a combinatorial multi-attribute, rich problem was either to frontally attack it, to address a simplified version, or to solve in a pipeline manner a series of simpler problems". Therefore, the constraints may be known also as attributes of the RVRP (VRP with multi-attributes).

More recently, [Rieck and Zimmermann 2010] state that: "Hence, research has turned to more specific and rich variants of the CVRP. The family of these problems is identified as rich vehicle routing problems. In order to model RVRPs, the basic CVRP must be extended by considering additional constraints or different objective functions". The evolution of models can be appreciated when new needs about the models itself emerge. On this respect, the authors stated: Rich vehicle routing problems are usually formulated as three-index vehicle-flow models with decision variables $x_{ij}^k$ which indicate whether an arc $(i,j) : i, j \in \Omega$ is traversed by vehicle $k$ ($k = 1, \ldots, K$). These models seem to be more flexible incorporating additional constraints, e.g., different capacities of the vehicles. In their monograph, [Toth and Vigo 2002] suggest that two-index vehicle-flow formulations *"generally are inadequate for more complex versions of vehicle routing problems"*. Their arguments are based on that *"these models are not suited for the cases where the cost or the feasibility of a circuit [each corresponding to a vehicle route] depends on the overall vertex sequence or on the type of vehicle allocated to the route"*. The new models have been extended to include other features in the logistic or supply chain process. Furthermore, [Schmid et al. 2013] have proposed

six integrative models considering the classical version of the VRP and some important extensions in the context of supply chain management. These extensions are lot-sizing, scheduling, packing, batching, inventory and intermodality. The authors state as benefit of their models that these consider an efficient use of resources as well as the inclusion of inter-dependencies among the subproblems. [Lahyani et al. 2012] have pointed out the importance of stating a common and closed definition for RVRP scope, "in most papers devoted to RVRPs, definitions of rich problems are quite vague and not significantly different. There is no formal definition either criterion which leads to decide whether or not a VRP is rich. Such definition has to rely on a relevant taxonomy which can help to differentiate among numerous variants of the VRP". In fact, the authors conclude their study with a numerical proposal for a specific definition: "a RVRP extends the academic variants of the VRP in the different decision levels by considering additional strategic and tactical aspects in the distribution system (4 or more) and including several daily restrictions related to the Problem Physical Characteristics (6 or more) [pure routing or operational]. Therefore, a RVRP is either a VRP that incorporates many strategic and tactical aspects and/or a VRP that reflects the complexities of the real-life context by various challenges revealed daily. The state of the art of RVRP has changed since 2006. Now studies incorporate *more complex* aspects of reality. Therefore, some variants described as rich by their authors in 2006 may not be considered as such anymore". So depending on the considered paper (or photography of achievements in research community), the RVRP definition will be evolving all the time.

In fact, some authors have stated that the taxonomy of VRPs is in constant evolution. The growing number of papers related to VRPs has created the necessity to classify the context and the different problems considered within. [Eksioglu et al. 2009] proposed a framework for classifying the literature of VRPs based on the scenario characteristics. They have tested it with a disparate set of VRP articles. So specific variants of VRP can be defined. However this study does not mention the emerging RVRPs. Recently, other realistic VRP variants have been promoted. The new VRP applications are expanding the scope of RVRP. Thanks to the technological advances, the Dynamic VRPs (so called real-time VRP) can be also considered as part of the wide Rich VRP scope [Pillac et al. 2013]. This branch includes the uncertainty over some variables (number of customers, travel times, and demands). Also it explodes the use of real-time communication of inputs (e.g., Global Positioning Systems). Therefore the target of this area is to generate 'good' routing solutions applicable to any change in the context and in a really fast way for each data variation. In general, the border of RVRPs with other VRP fields is blurred. Other emerging VRP variants can be included inside of RVRP for its current interest and future impact. One highlighted application is the Green VRP [Lin et al. 2014; Erdoğan and Miller-Hooks 2012]. On this sustainable transportation issues are involved though objective functions or variables related to environmental costs. Inside of this a special branch have been developed as the Pollution VRP [Bektaş and Laporte 2011; Demir et al. 2012a,b]. Its main objective is to reduce gasses emission on transportation activities. The combination of previous VRP branches represent promising applications of RVRP, as it has been recently promoted in a Special Issue of Rich and Real-Life VRPs [Juan et al. 2014].

To sum up, we could conclude that a Rich VRP reflects, as a model, most of the relevant attributes of a real-life vehicle-routing distribution system. These attributes might include several of the following: dynamism, stochasticity, heterogeneity, multi-periodicity, integration with other related activities (e.g., vehicle packing, inventory management, etc.), diversity of users and policies, legal and contractual issues, environmental issues, etc. Thus, as a model, a Rich VRP is an accurate representation of

a real-life distribution system and, therefore, the solutions obtained for the Rich VRP should be able to be directly applied to the real-life scenario.

As it can be appreciated, the implications of the RVRP definition has evolved to a more close concept during time. The new demanding needs of enterprises have forced to consider more complex approaches. There is also a clear trend of creating generic and efficient approaches. Considering the large number of papers that have been devoted to the VRP, just a few of these could be applied to the current RVRP context. There are a small number of papers that have explicitly addressed the RVRP. This fact emphasises the emptiness in the literature as well as the opportunities that the academy sector has to collaborate with enterprises addressing real routing problems. Next section presents a literature review on some strategies aimed at solving Rich VRP instances with more than one constraint simultaneously.

## 5. LITERATURE REVIEW

In this section, we can find more than 50 papers selected because they are denominated *Rich* extensions of the original VRP or are related to other RVRPs, plus some few others that consider several VRP variants. They have in common that they consider one or more variants of the classical VRP. The approaches presented on these papers solve separated VRP variants or with different combinations of their constraints. One of the first-explicitly RVRP cases is presented in [Pellegrini 2005]. The author addresses a specific RVRP approach with the consideration of heterogeneous fleet, multiple time windows, the delivery cannot be offered in some intervals of time and there is a maximum time for a single tour. They proposed two heuristic algorithms based on the well-known Nearest Neighbour (NN) heuristic procedure [Solomon 1987] combined with a swap local search. In this article, a Deterministic version of a NN (DNN) algorithm as well as a Randomised NN (RNN) version is created. A random behaviour to the selection of the next customer in the building process of a route is added to the procedure. The author showed encouraging results in a short computational time with generated instances of 50, 100, 150, and 200 customers. The RNN algorithm reaches better results than the DNN version. Although the RNN version losses some efficiency as the number of customers increases.

On the other hand, [Goel and Gruhn 2005, 2006] address the capacity restrictions, time windows, heterogeneous fleet with different travel times, and also multiple pickup and delivery locations, travel costs, different start and end locations for vehicles and other constraints. They propose iterative improvement approaches based on LNS. The authors have created an instance generator of 50, 100, 250 and 500 orders to show the performance of their approach. Likely, [Goel and Gruhn 2008] consider other set of real-life requirements —e.g., time window restrictions, a heterogeneous vehicle fleet with different travel times, travel costs and capacity, multi-dimensional capacity constraints, order/vehicle compatibility constraints, orders with multiple pickup, delivery and service locations, different start and end locations for vehicles, and route restrictions for vehicles. The authors propose an iterative improvement approach. They use a reduced Variable Neighbourhood Search (VNS) algorithm for exchanging elements between neighbourhood, and also a LNS approach for using nested neighbourhoods of different size. This combination helps to avoid local minimum.

Following the LNS research line, [Ropke and Pisinger 2006b,a] propose a heuristic based on LNS as proposed by [Shaw 1998]. Furthermore, their approach is a unified heuristic with an adaptive layer. They are focused on the VRP with backhauls (VRPB) with time windows, pickup-and-delivery and multi-depots. They propose a model transformation of the VRPB to solve the simultaneous pickup-and-delivery. Nine data sets are used to test several configurations of the proposed heuristic, where more of the 50% of best known solutions for those instances are improved. Later,

the same authors developed an Adaptive LNS framework [Pisinger and Ropke 2007] for addressing the capacitated, time windows, multi-depot, split-deliveries and open routes constraints. They use several sets of instances with up to 1000 customers, and improve 183 best known solutions out of 486 benchmark tests.

[Hasle et al. 2005] shortly describe four mechanisms to enhance scalability and present a generic route construction heuristic for RVRPs. The empirical investigation results based on standard test instances for several VRP variants show the effectiveness of this approach. Likely, [Hasle and Kloster 2007] propose a generic approach to harness modelling flexibility. The authors present a generic solver based on a unified algorithmic approach which is a combined operation of the Variable Neighbourhood Descent and a promising Iterated Local Search (ILS) [Lourenço et al. 2010]. An initial solution is generated using the parallel version of CWS. They address the capacitated constraint, the distance limitation, the pickup-and-delivery, the fleet size and mix problem as well as the time windows. They present the possibility to extend it for multi-depot and site-dependent problems. Classical benchmarks of [Solomon 1987] and their modification of [Li and Lim 2001] are also used. Their results are based on a range of customers between 50 up to 1000.

A wide classification of the RVRP variants is presented in a special issue published by [Hartl et al. 2006]. Seven papers were selected for covering different aspects and illustrating novel types of VRP applications. The editors state "VRP research has often been criticised for being too focused on idealised models with non-realistic assumptions for practical applications". Several optimisation methods are proposed for solving problems inspired in real applications of VRP knowledge. For instance, [Reimann and Ulrich 2006] addressed the VRP with backhauls and time windows. [Hoff and Løkketangen 2006] is focused in the Travelling Salesman Problem with pickup and delivery. [Ileri et al. 2006] work in the pickup and delivery requests with time windows, heterogeneous fleet, and some operational constraints over the driver routes. The authors use a Set Partitioning technique and also Column Generation to solve real-life instances. [Fügenschuh 2006] proposes a metaheuristic for the VRP with coupling time windows. This method combines classical construction aspects with mixed-integer preprocessing techniques and it is improved with a randomised search strategy. Several randomly generated instances are used, as well as a real-world case for public bus transportation considering school times in rural areas of Germany. [Magalhães and Sousa 2006] present a real case adopting a system of variable routes that are dynamically designed. [Sörensen 2006] shows a bi-objective case considering marketing and financial interests for being solved using metaheuristic. [Bolduc et al. 2006] addressed a multiple period horizon in an inventory context with heterogeneous fleet, multi-trips, and capacity restrictions. The authors use heuristics to minimise the cost of distributing products to the retailers and the cost of maintaining inventory at the facility. Randomly generated instances were used to measure the performance of the approach with two sets of small and large cases.

[Pellegrini et al. 2007] have presented a case study characterised by multiple objectives, constraints concerning multiple time windows, heterogeneous fleet of vehicles, maximum duration of the sub-tours, and periodic visits to the customers. They considered two versions of ACO: (a) Multiple Ant Colony System (M-ACS) first proposed by [Dorigo and Gambardella 1997]; and (b) MAX-MIN Ant System (MMAS) based on [Stützle and Hoos 1997]. The authors compared the results with a Tabu Search (TS) algorithm and a Randomised NN (RNN) heuristic which was mentioned before. Both ACO algorithms perform significantly better than the TS and RNN approaches, using an instance generator of 70-80 orders. Other ACO implementation is proposed by [Rizzoli et al. 2007] which has been applied to real contexts addressing separately heterogeneous fleet, time windows, pickup and delivery, and time dependent. The authors

have tested four ACO algorithms using data from real distribution companies between 15 and 600 customers.

In [Hoff 2006], we can find four papers [Hoff and Løkketangen 2006, 2007; Hoff et al. 2009, 2010] focused in the development of Lasso Solution Strategies using TS and heuristics for the VRP with pickup and delivery, time-depending and stochastic demands. Lasso Strategies consists on a path or spoke that is first followed by each vehicle to perform deliveries, the remaining customers assigned to this vehicle are then visited along a loop and, finally, the spoke is followed in the reverse order to perform pickups. If the loop is empty, then the lasso reduces to a double-path; if the path is empty, then it reduces to a Hamiltonian cycle.The authors have created instances with 7–262 nodes which are derived from classical benchmarks used in CVRP. A real-life problem from a Norwegian company is also considered. In [Derigs and Döhmer 2005], the authors also addressed the pickup and delivery VRP with time windows. They proposed an indirect search procedure based on sequence/permutation of tasks, cheapest insertion of a visit, and a Threshold-Accepting like a local search metaheuristic. The proposed algorithm was implemented into a Decision Support System for a removal firm. They produce some promising preliminary results with randomly generated instances.

[Irnich 2008] takes advantage of strong modelling capabilities and proposes a Unified Modelling and Heuristic Solution Framework. The author highlight the potential of $k - edge$ exchange neighbourhoods. This approach is intended to support efficient local search procedures for addressing all standard types of VRPs, such as the capacitated and distance-constrained, multiple depots, time windows, simultaneous delivery and pickup, backhauling, pickup-and-delivery problems, periodic VRP, fleet mix and size, site dependencies as well as mixtures and extensions of these. The author proposes to integrate the efficient search blocks into different metaheuristic. Some promising results are presented for VRPTW and MDVRPTW combining a VNS with LNS strategies —inspired on the work of [Ropke and Pisinger 2006b].

There is a large number of studies using exact methods or combinations of them. In [Wen 2010], we can find three papers that address some variants of the Rich VRP inspired in real-life situations. The author proposes different strategies to solve each: (a) the VRP with cross-docking options through a TS based heuristic tested over 200 pairs of suppliers and customers [Wen et al. 2008]; (b) the dynamic VRP with multiple objectives over a planning horizon that consists of multiple periods through MILP and a three phase heuristic [Wen et al. 2010]; and (c), the VRP with multi-period horizon, time windows for the delivery, heterogeneous vehicles, drivers working regulations, and other constraints [Wen et al. 2011]. In the last work, the author proposes a MILP embedded by a multilevel VNS algorithm. Good quality solutions for solving up to 2000 orders are generated using a real case information. In this same research line, [Rieck and Zimmermann 2010] propose a new MILP (two-index vehicle-flow) model for a Rich VRP with docking constraints. They consider time windows, simultaneous delivery and pick-up at customer locations and multiple uses of vehicles. The test instances with 10-30 customers were generated from the classical set of VRP with Time Windows [Solomon 1987]. The proposed method solves small and medium problem instances efficiently. Other promising approach, as proposed by [Doerner and Schmid 2010], consists in the combination of exact algorithms and metaheuristic search components. The authors present a survey of several hybrid techniques and also highlights some key aspects for future studies. Hybrid approaches allow conquering the obstacles observed when the individual concepts are applied independently. They present three trends of hybridisation schemes: set-covering based, local branching approaches, and decomposition techniques. They addressed the periodic VRP with time windows and the multi-depot VRP with time windows, but other variants are commented. An exact

solution framework based on Set Partitioning (SP) modelling is proposed by [Baldacci et al. 2010; Baldacci et al. 2011; Baldacci et al. 2011] for individual types of VRPs. The results outperform all other exact methods published so far and also solve several previously unsolved test instances. The preliminary step to the proposed framework is presented on [Baldacci and Mingozzi 2009] where a unified exact method based on set partitioning is introduced for solving the well-known CVRP, HVRP, SVRP and the MDVRP. Computational results asses the performance of their approach over the main instances from the literature of the different variants of HVRP, SVRP and MDVRP.

Several studies have developed Column Generation-based (CG) methods as well. [Oppen et al. 2010] consider a real scenario called the Livestock Collection Problem (LCP) which is considered a Rich VRP extended with inventory constraints. This context includes duration and capacity restrictions, heterogeneous fleets, time windows, multi-trips, and multi-products issues. The authors addressed it through an exact solution method based on CG. The authors have created instances with less than 30 customers' orders inspired in real-world. The CG approach has helped to find optimal solutions in different scenarios. But the authors defined limitations for finding optimal solutions to LCP instances. Another CG heuristic is proposed by [Goel 2010] for addressing a VRP with time windows, heterogeneous vehicle fleet, multiple depots, and pickup-and-delivery. Some small instances are randomly generated in order to test the heuristic performance. [Ceselli et al. 2009] also propose the use of a CG combined with a dynamic programming algorithm in order to address simultaneously a heterogeneous fleet, different depots, time windows, route length, optionally opened routes, pickup and delivery and several other constraints. The authors tested their approach with 46 randomly generated instances composed by 100 orders and the results are compared with valid lower bounds. Under a similar restricted context, [Ruinelli 2011] has compared three methods on a master thesis: an Ant Colony System (ACS), a CG algorithm and a general purpose MILP solver. Computational results are presented using 14 real instances from a distribution company, where the CG outperforms the other two methods. [Prescott-Gagnon et al. 2012] present a real-life case of an oil distribution which presents a set of particular features. Some of the constraints addressed are the heterogeneous vehicle fleet, multiple depots, intra-route replenishment, time windows, driver shifts and optional customers. The authors propose three metaheuristic, namely, TS algorithm, a LNS heuristic combined with TS , and another LNS based on a CG heuristic. Computational results indicate that both LNS methods outperform the TS heuristic. In fact, the LNS method based on CG tends to produce better quality solutions. Also [Lannez et al. 2010] present an approach based on CG for a very particular extension of Rich VRP called Rich Arc Routing Problem, where the demand is located on the arcs and not in the nodes.

Other generic Rich solvers have emerged in the literature. [Cordeau et al. 1997, 2001; Cordeau and Laporte 2003; Cordeau et al. 2004] propose a Unified Tabu Search approach for VRPs with time windows, multi-period, multi-depot, and site-dependent. Several real and theoretical benchmarks have been used to test the performance of this approach. Some ILS approaches are proposed by [Ibaraki et al. 2005; Hashimoto et al. 2006, 2008]. In fact, [Subramanian 2012] propose a promising combination of ILS with Integer Programming aspects for several VRP variants. This work was extended to the Fleet Size and Mix (FSM) and HVRP research line in [Subramanian et al. 2012]. They have developed a hybrid algorithm composed by an ILS based heuristic and a Set Partitioning (SP) formulation. The SP model is solved by using a MIP solver that calls the ILS heuristic during its execution. Three benchmark instances with up to 360 customers were used to test the approach. For instance, [Groër et al. 2010] implemented a library of 7 local search heuristics for addressing several variants like the CVRP, VRPTW and MDVRP. Some classical heuristic are used —e.g., Record-to-

Record, CWS. Their approach is based on easily remove and insert customers from an existing solution (called neighbourhood ejection). Several classical benchmarks are used to show the performance of their approach. In [Battarra 2011] several exact and heuristic algorithms for routing problems are presented in individual Rich VRP cases [Baldacci et al. 2009; Battarra et al. 2009]. Some of the problems addressed are the FSM and the HVRP with multi-trips and time windows.

Recently, [Santillán et al. 2012] solve a Routing-Scheduling-Loading using a heuristic-based system. As first step, the proposed system applies an ACS for the Routing and Scheduling Problem, then a Bin Packing technique is used for the Vehicle Load problem. Some tests with [Solomon 1987] instances are developed. Also the authors use real information from the distribution of bottles provided by a Mexican company. Another hybrid approach is proposed by [Vallejo et al. 2012]. They apply a three-phase heuristic which merges the use of a memory-based approach with clustering techniques. The authors present promising test results using between 100 and 2000 customers comparing their approach against a Genetic Algorithm. Next two particular real cases are presented, inspired on [Ropke and Pisinger 2006b]. First, [Amorim et al. 2012] create a new Adaptive LNS for solving specific real instances of a heterogeneous fleet site dependent vehicle routing problem with multiple time windows. This case is inspired in a food distribution company in Portugal. Second, [Derigs et al. 2013] propose to combine the commented ALNS with Local Searches both controlled by two metaheuristic procedures (Record-to-Record travel and attribute based Hill Climber) for addressing a particular real case called Rollon-Rolloff VRP (RRVRP) occurred in sanitation/waste collection.

[Vidal et al. 2013a] develop a study over 64 metaheuristic comparing their solutions on 15 classic variants of VRP with multi-attributes. They present a classification on the types of constraints as attributes and identify promising principles in algorithmic-designing for Rich VRP. In fact, they state that the critical factors for efficient metaheuristic is the appropriate balance between intensification and diversification explorations in the solution space. The authors conclude that the combination of hybrid algorithms and parallel cooperative methods would create effective solvers. Later the same authors proposed a unified solution framework called Unified Hybrid Genetic Search (UHGS) for several types of Rich VRP [Vidal et al. 2013c]. The framework uses efficient generic local search and genetic operators. This approach is also based on a giant-tour representation with a split procedure originally proposed by [Prins 2004]. The authors present interesting computational results using 39 benchmarks over 26 different Rich VRPs. Furthermore, the authors apply their method combined with diversity management mechanisms to different large scale instances of Rich Time-constrained VRPs [Vidal et al. 2013b]. The used instances involve up to 1000 customers. The proposed framework outperforms all current state-of-the-art approaches. This is addressed to any combination of periodic, multi-depot, site-dependent, and duration-constrained VRP with time windows.

In Table I a summary of the cited state-of-the-art approaches developed for the Rich VRP is presented by authors, year of publication, type of proposed method, and maximum number of customers addressed in the study. As the reader can appreciate the rows are sorted by type of method, year and last name of first author. Also we have applied a restrictive filter if the approach can solve more than one Rich VRP. The star (*) on the last column highlights the approaches that have been or can be tested with no restriction on the combination of constraints. The table is divided in two parts: complete methods first and incomplete later.

Table I. State-of-the-art of Rich VRP methods.

| Authors | Year | Method | Maximum $n$ | Several Rich VRPs |
|---|---|---|---|---|
| Ruinelli | [2011] | Column Generation | 150 | |
| Baldacci et al. | [2011] | Exact Method | 200 | √* |
| Baldacci et al. | [2011] | Exact Method | 200 | √* |
| Baldacci et al. | [2010] | Exact-Solution Framework | 200 | √* |
| Bettinelli et al. | [2011] | Branch-and-Cut-and-Price | 144 | |
| Doerner and Schmid | [2010] | MatHeuristics | - | |
| Goel | [2010] | Column Generation | 250 | |
| Oppen et al. | [2010] | Column Generation | 27 | |
| Rieck and Zimmermann | [2010] | Mixed-Integer Linear Programming | 30 | |
| Baldacci and Mingozzi | [2009] | Set Partitioning | 100 | √ |
| Ceselli et al. | [2009] | Column Generation | 100 | |
| Fügenschuh | [2006] | Mixed-Integer Programming | 404 | |
| Derigs et al. | [2013] | LS/LNS-based metaheuristic | 199 | |
| Vidal et al. | [2013b] | Hybrid Genetic Search with Advanced Diversity Control | 1000 | √* |
| Amorim et al. | [2012] | Adaptive Large Neighbourhood Search Framework | 366 | |
| Santillán et al. | [2012] | Ant Colony System | 100 | |
| Subramanian et al. | [2012] | Iterated Local Search | 360 | |
| Vidal et al. | [2013c] | Unified local search and Hybrid Genetic Search | 480 | √* |
| Vallejo et al. | [2012] | 3-phase heuristic using a memory-based and clustering techniques | 2000 | |
| Battarra | [2011] | Exact and Heuristic algorithms | 100 | √ |
| Groër et al. | [2010] | Local Search Heuristic | 483 | |
| Prescott-Gagnon et al. | [2012] | Tabu Search, LNS+TS heuristic, LNS+CG heuristic | 750 | |
| Wen et al. | [2010] | 3-phase heuristic | 80 | |
| Goel and Gruhn | [2008] | Variable and Large Neighbourhood Searches | 40 | |
| Irnich | [2008] | Heuristic Framework using Local Search-Based metaheuristic | 1000 | √* |
| Wen et al. | [2008] | TS and Adaptive Memory Procedure | 200 | |
| Hasle and Kloster | [2007] | metaheuristic | 199 | √* |
| Pellegrini et al. | [2007] | Multiple Ant Colony Optimisation | 80 | |
| Pisinger and Ropke | [2007] | LNS Heuristic | 1008 | √* |
| Rizzoli et al. | [2007] | Ant Colony Optimisation | 600 | √ |
| Bolduc et al. | [2006] | Heuristics | 75 | |
| Goel and Gruhn | [2006] | Large Neighbourhood Search | 500 | |
| Hoff and Løkketangen | [2006] | Tabu Search Heuristic | 262 | |
| Ileri et al. | [2006] | Set partitioning model | 130 | |
| Magalhães and Sousa | [2006] | Clustering Heuristic | 450 | |
| Reimann and Ulrich | [2006] | Ant Colony Optimisation | 100 | |
| Ropke and Pisinger | [2006b] | LNS Heuristic | 500 | √ |
| Ropke and Pisinger | [2006a] | LNS Heuristic | 500 | √ |
| Sörensen | [2006] | Memetic algorithm with population management | 199 | |
| Derigs and Döhmer | [2005] | Local Search Algorithm | - | |
| Goel and Gruhn | [2005] | Large Neighbourhood Search | 500 | |
| Pellegrini | [2005] | Nearest Neighbour | 200 | |
| Cordeau et al. | [2004] | Improved Unified Tabu Search heuristic | 288 | √ |
| Cordeau et al. | [2001] | Unified Tabu Search heuristic | 1035 | √ |
| Cordeau et al. | [1997] | Tabu Search | 288 | |

## 6. CLASSIFICATION OF RICH VRP PAPERS

Most of the routing constraints considered in the previous works were unified and classified. The next list presents the main distribution constraints considered on these papers. Table II shows the presence of each constraint on the cited papers. This is useful to appreciate the diversity of cataloged papers as Rich VRPs. And finally, in Table III a classification of these routing constraints is done using the cited studies of [Vidal et al. 2013c; Lahyani et al. 2012]. In [Vidal et al. 2013c], the routing constraints are related to its representation point inside of the inner methodology process. For this, they propose three groups which represent the simple aspects that any solver must deal with: Assignment of customers and routes to resources, the Sequence choices, and

the Evaluation of fixed sequences. The authors state that this "simple classification is intimately connected with the resolution methodology". In [Lahyani et al. 2012], constraints are associated to the company decision levels (operational, tactical, and strategical). The first level (strategic) includes decisions related to the locations, the number of depots used and, the data type. The tactical level defines the order type and the visit frequencies of customers over a given time horizon. Finally, the operational considers vehicle and driver schedules so the constraints are related to the distribution planning and specified for customers, vehicles, drivers, and roads. Additionally, we propose a second level of classification associated to the routing element involved (depot, customer, route, vehicle, and product) in order to help for a better understanding of the classification.

— Multi-Products (**CP**): Some vehicles can carry out several types of products (fresh-cold, small-big, etc.).
— Multi-Dimensional capacity (**CD**): The capacity of vehicles is considered in 2D or 3D.
— Vehicle Capacity (**C**): The capacity of vehicles is limited.
— Homogeneous Fleet of Vehicles (**FO**): All vehicles of the fleet have the same capacity.
— Heterogeneous Fleet of Vehicles (**FE**): Several type of vehicles (capacities) can be found in the fleet.
— Unfixed Fleet of Vehicles (**VU**): The number of vehicles considered is unlimited.
— Fixed Fleet of Vehicles (**VF**): The number of vehicles considered is limited.
— Fixed Cost per Vehicle (**FC**): To use a vehicle implies an extra cost.
— Variable Cost of Vehicle (**VC**): The real cost is represented by the product of the distance assigned to a vehicle and its price per distance unit.
— Multi-Trips (**MT**): All or some vehicles of the fleet can execute more than one trip (multiple uses of vehicles).
— Vehicle Site Dependent (**DS**): Some vehicles can not visit some nodes due to geographical, compatibility or legal issues.
— Vehicle Road Dependent (**DR**): Some vehicles can not pass through some edges of the network for some legal issues.
— Duration Constraints/Length (**L**): The duration of each route cannot exceeded a maximum value or cost, including service times on each visited client.
— Driver Shifts/Working Regulations (**D**): The design of routes include the number of legal working hours of drivers (stops, breaks, rest, etc).
— Balanced Routes (**BR**): The load of routes or vehicles must be balanced between all.
— Symmetric Cost Matrix (**CS**): The cost matrix has a symmetric nature.
— Asymmetric Cost Matrix (**CA**): The cost matrix has an asymmetric nature.
— Intra-route replenishments (**IR**): The vehicles must be re-loaded in some point of the routes.
— Time Dependent/Dynamic/Stochastic times (**TD**): The target is minimising time and the travelling times could vary during a day (hard or flexible). The location/distance of clients changes.
— Stochastic Demands/Dynamic (**S**): The demands of clients can change during the application of a routing solution.
— Time Windows (**TW**): The clients can not receive the orders out of a time windows. Each client has a particular time window (hard or soft).
— Multiple Time Windows (**MW**): The clients can not receive the orders out of a set of time windows. Each client has a particular set of time windows.
— Pick-up & Delivery (**PD**): The construction of routes must consider the picking up of products in some clients and the delivery to others, in a sequential or separate way. The depot just define the starting/ending point of vehicles.

— Simultaneous Pick-up & Delivery (**PS**): The construction of routes must consider the picking up and delivery of products/persons at the same time in all nodes by the same vehicle. The depot just defines the starting/ending point of vehicles.

— Backhauls (**B**): The construction of routes must consider the picking up of products in some clients and the delivery to others, in a sequential or separate way. The critical assumption is that all deliveries must be made on each route before any pickups can be made (sometimes a client could require both a delivery and a pick-up). The re-arrangement of products could be expensive or unfeasible. The depot just define the starting/ending point of vehicles.

— Multiple Visits/Split deliveries (**MV**): The clients are visited several times for delivering the summary of the original orders. Each vehicle may deliver a fraction of a customer's demand.

— Multi-Period/Periodic (**MP**): The optimisation is made over a set of days, considering several visits and each client has a different frequency of visits.

— Inventory Levels Controls (**I**): The costs of stocks are also considered to be minimised with the routing costs while the levels of stock are controlled.

— Customer Capacity (**CC**): The capacity stock of clients is also considered.

— Multi-Depot (**MD**): There are more than one depot from where the vehicles leave and arrive.

— Time Windows for the Depot (**WD**): The depot is open during a period of time. So if vehicles need to do more than one trip need to consider this.

— Different end locations/Open Routes (**O**): The routes start at the depot but finish on the last client. The return cost is not considered (optional).

— Different start and end locations (**DA**): The vehicles start and end in different locations.

— Departure from different locations (**DD**): The vehicles start in different locations.

— Precedence constraints (**PC**): The visiting order of clients could be important for the loading and unloading of products. Its order could be important for healthy or security reasons.

— Multi-Objectives (**MO**): The study consider more than one objective function or related costs at the same time.

Table II. Relation of Rich VRP papers and considered constraints.

| Authors | CP | CD | C | FO | FE | VU | VF | FC | VC | MT | DS | DR | L | D | BR | CS | CA | IR | TD | S | TW | MW | PD | PS | B | MV | MP | I | CC | MD | WD | O | DA | DD | PC | MO |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| [Derigs et al. 2013] | | | ✓ | ✓ | ✓ | ✓ | | | | | | | | | | ✓ | | | | | | | ✓ | | | | ✓ | | ✓ | | | | | | | |
| [Vidal et al. 2013b] | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | ✓ | | | | ✓ | ✓ | ✓ | ✓ | | | | | | ✓ | | ✓ | ✓ | | | | ✓ |
| [Amorim et al. 2012] | | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | ✓ | ✓ | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | ✓ | | ✓ | | | | | |
| [Santillán et al. 2012] | | | ✓ | | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | | | | | ✓ | | | | | ✓ | ✓ | | | | | | | ✓ | | ✓ | | | | | ✓ |
| [Subramanian 2012] | | | | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | | | | | ✓ | | | | | ✓ | ✓ | | | | | | | ✓ | | ✓ | | | | | |
| [Subramanian et al. 2012] | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | | | | ✓ | | | | | ✓ | | | ✓ | | | ✓ | | ✓ | | ✓ | | | | | |
| [Vidal et al. 2013c] | | | ✓ | | ✓ | ✓ | | | | | | | | | | ✓ | | | | | ✓ | | | | | | | | ✓ | | | | | | | |
| [Vallejo et al. 2012] | | | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | | | | | ✓ | | | | | ✓ | | | | | | | | ✓ | | | | | | | |
| [Baldacci et al. 2011] | | | ✓ | ✓ | ✓ | | | | | ✓ | | | | | | ✓ | | | | | ✓ | | ✓ | | ✓ | | ✓ | | ✓ | | | | | | | ✓ |
| [Baldacci et al. 2011] | | | ✓ | ✓ | ✓ | | | | | | | | | | | ✓ | | | | | | | | | | | | | ✓ | | | | | | | |
| [Battarra 2011] | | | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | | | ✓ | | ✓ | | | | | ✓ | ✓ | ✓ | | ✓ | | ✓ | | ✓ | | ✓ | ✓ | | | | |
| [Bettinelli et al. 2011] | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | | ✓ | | | ✓ | | | | | ✓ | | ✓ | | | | ✓ | | ✓ | | | ✓ | ✓ | | | |
| [Ruinelli 2011] | | | ✓ | | | ✓ | ✓ | | | | ✓ | | | | | ✓ | | | | | ✓ | | ✓ | | | | ✓ | | ✓ | | | | | | | |
| [Yu and Zhen-Yang 2011] | | | ✓ | | | ✓ | | | | | | | | | | ✓ | | | | | ✓ | | | | | | | | | | | | | | | |
| [Baldacci et al. 2010] | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | | | | | ✓ | | | | ✓ | | | | | | ✓ | | | | | |
| [Doerner and Schmid 2010] | | | ✓ | ✓ | ✓ | ✓ | ✓ | | | | ✓ | | ✓ | ✓ | | ✓ | | | | | ✓ | | ✓ | | | | ✓ | | ✓ | | | | | | | |
| [Groër et al. 2010] | | | ✓ | | | | | | | ✓ | | | | | | ✓ | | | | | ✓ | | | | | | | | ✓ | | | | | | | |
| [Oppen et al. 2010] | ✓ | | ✓ | ✓ | ✓ | ✓ | | | | | | ✓ | ✓ | ✓ | | ✓ | | | | | ✓ | | | ✓ | ✓ | | | | | ✓ | | | | | | |
| [Prescott-Gagnon et al. 2012] | | | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | | ✓ | ✓ | | ✓ | | ✓ | | | ✓ | | | | | | ✓ | ✓ | ✓ | ✓ | | | | | | |
| [Wen et al. 2010] | | | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | | | ✓ | | ✓ | | | | ✓ | ✓ | | | | ✓ | | ✓ | ✓ | ✓ | | ✓ | ✓ | | ✓ | | ✓ |
| [Rieck and Zimmermann 2010] | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| [Goel 2010] | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | | | ✓ | | | | ✓ | ✓ | | ✓ | ✓ | | ✓ | | | ✓ | | ✓ | ✓ | ✓ | | | |
| [Ceselli et al. 2009] | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | ✓ | | ✓ | | | | ✓ | ✓ | | ✓ | | | ✓ | | | ✓ | | ✓ | ✓ | ✓ | | | |
| [Baldacci and Mingozzi 2009] | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | ✓ | | | | | ✓ | | ✓ | | | ✓ | | | ✓ | | ✓ | | | | | ✓ |
| [Crainic et al. 2009a] | | | ✓ | ✓ | | ✓ | | | | | | | ✓ | | | ✓ | ✓ | | | | ✓ | ✓ | | | | | | | ✓ | | ✓ | | | | | |
| [Goel and Gruhn 2008] | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | ✓ | | ✓ | | | ✓ | | | ✓ | | ✓ | ✓ | | | | ✓ | | | | | | | | | | |
| [Hashimoto et al. 2008] | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | | ✓ | ✓ | | ✓ | | | | | ✓ | | ✓ | | | | | | ✓ | | ✓ | ✓ | | | | |
| [Irnich 2008] | | | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | | ✓ | | | | | ✓ | | | | | ✓ | ✓ | ✓ | | ✓ | | | ✓ | ✓ | | | | | | | |
| [Wen et al. 2008] | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | ✓ | | ✓ | | | ✓ | | | | | ✓ | | ✓ | | | ✓ | | | ✓ | | | | | | | |
| [Hasle and Kloster 2007] | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | ✓ | | | | | ✓ | | | | | ✓ | | ✓ | | | | | | | | | | | | | ✓ |
| [Pellegrini et al. 2007] | | | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | | ✓ | | | ✓ | | | | | ✓ | | ✓ | | ✓ | | | | | | ✓ | | | | | |
| [Pisinger and Ropke 2007] | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | ✓ | | | | | ✓ | | | | | ✓ | | ✓ | | | | | | ✓ | | ✓ | | | | | |
| [Rizzoli et al. 2007] | | | ✓ | ✓ | ✓ | ✓ | | | | | | | | | | ✓ | ✓ | | | | ✓ | ✓ | | | | ✓ | ✓ | | | | | | | | | |
| [Bolduc et al. 2006] | | | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | | | | | ✓ | | | | | | | | | | ✓ | | | | | | | | | | |
| [Fügenschuh 2006] | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | ✓ | | | | | ✓ | ✓ | ✓ | | | | | | | | | | | | | |
| [Hashimoto et al. 2006] | | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | | ✓ | ✓ | | ✓ | | | ✓ | | ✓ | | | ✓ | ✓ | ✓ | | | | | | ✓ | | ✓ | | | | | ✓ |
| [Goel and Gruhn 2006] | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | ✓ | | | ✓ | | | | | ✓ | | ✓ | | | | | | | | | | | | | |
| [Hoff and Løkketangen 2006] | | | ✓ | ✓ | | ✓ | | | | | | | | | | ✓ | | | | | | | | | | | | | | | | | | | | |
| [Ileri et al. 2006] | | | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | | ✓ | | | ✓ | | | | | ✓ | | ✓ | | | ✓ | | | ✓ | | ✓ | | | | | |
| [Magalhães and Sousa 2006] | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | | | | | ✓ | | | | | ✓ | | | | | | | | | | | | | | | |
| [Reimann and Ulrich 2006] | | | ✓ | ✓ | ✓ | ✓ | | | | | | | | | | ✓ | | | | | ✓ | | | ✓ | | | | | | | | | | | | |
| [Ropke and Pisinger 2006b] | | | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | | | | | ✓ | | | | | ✓ | | | ✓ | | | | | ✓ | | | | ✓ | | | ✓ |
| [Ropke and Pisinger 2006a] | | | ✓ | ✓ | ✓ | ✓ | ✓ | | | | ✓ | | ✓ | | | ✓ | | | | | ✓ | | | | | ✓ | | | | | | | | ✓ | | |
| [Sörensen 2006] | | | ✓ | | ✓ | ✓ | | | | | | | ✓ | | | | | | | | | | | | | | ✓ | | | | | | | | | |
| [Derigs and Döhmer 2005] | | | ✓ | ✓ | | ✓ | ✓ | ✓ | | ✓ | ✓ | | | | | ✓ | | | | ✓ | ✓ | ✓ | ✓ | | | | | | | | ✓ | | | | | |
| [Goel and Gruhn 2005] | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | ✓ | | | ✓ | | | | | ✓ | ✓ | | | | ✓ | | | | | | | | | | |
| [Ibaraki et al. 2005] | ✓ | | ✓ | ✓ | ✓ | ✓ | | | | | | | | | | ✓ | | | | ✓ | ✓ | | | | | | | | ✓ | | ✓ | | | | | |
| [Pellegrini 2005] | | | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | | ✓ | | | ✓ | | | | | ✓ | | | | | | ✓ | | | | | | | | | |
| [Cordeau et al. 2004] | | | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | | | | | | ✓ | | | | | ✓ | ✓ | | | | ✓ | | | ✓ | | | | | | | |
| [Cordeau and Laporte 2003] | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | | | ✓ | | | ✓ | | | | ✓ | ✓ | ✓ | | | | | | | | | | | | | | |
| [Cordeau et al. 2001] | | | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | | | | | ✓ | | | | | ✓ | | ✓ | | | ✓ | | | ✓ | | ✓ | | | | | |
| [Cordeau et al. 1997] | | | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | | | | ✓ | ✓ | | | | | ✓ | ✓ | | | | ✓ | | | ✓ | | ✓ | | | | | ✓ |

Table III. Classification of main documented Rich VRP constraints

| Restriction | Code/Id | [Vidal et al. 2013c] Classification | [Lahyani et al. 2012] Classification | Our 2nd Level Classification |
|---|---|---|---|---|
| Multi-Products | CP | Assign | Strategic | Veh-Prod |
| Multi-Dimensional capacity | CD | Assign | Strategic | Veh-Prod |
| Vehicle Capacity | C | Assign | Operational | Veh |
| Homogeneous Fleet of Vehicles | FO | Assign | Operational | Veh |
| Heterogeneous Fleet of Vehicles | FE | Assign | Operational | Veh |
| Unfixed Fleet of Vehicles | VU | Evaluation | Operational | Veh |
| Fixed Fleet of Vehicles | VF | Assign | Operational | Veh |
| Fixed Cost per Vehicle | FC | Evaluation | Operational | Veh |
| Variable Cost of Vehicle | VC | Evaluation | Operational | Veh |
| Multi-Trips | MT | Sequence | Operational | Veh |
| Vehicle Site Dependent | DS | Assign | Operational | Veh-Cust |
| Vehicle Road Dependent | DR | Assign | Operational | Veh-Route |
| Duration Constraints/Length | L | Evaluation | Operational | Route-Driver |
| Driver Shifts/Working Regulations | D | Evaluation | Operational | Route-Driver |
| Balanced Routes | BR | Assign | Operational | Route-Driver |
| Symmetric Cost Matrix | CS | Sequence | Operational | Route |
| Asymmetric Cost Matrix | CA | Sequence | Operational | Route |
| Intra-route replenishments | IR | Assign | Tactical | Route |
| Time Dependent/Dynamic/Stochastic times | TD | Evaluation | Tactical | Route |
| Stochastic Demands/Dynamic | S | Evaluation | Tactical | Customer |
| Time Windows | TW | Evaluation | Tactical | Customer |
| Multiple Time Windows | MW | Evaluation | Tactical | Customer |
| Pick-up & Delivery | PD | Sequence | Tactical | Customer |
| Simultaneous Pick-up & Delivery | PS | Evaluation | Tactical | Customer |
| Backhauls | B | Sequence | Tactical | Customer |
| Multiple Visits/Split deliveries | MV | Assign | Tactical | Customer |
| Multi-Period/Periodic | MP | Assign | Tactical | Customer |
| Inventory Levels Controls | I | Assign | Tactical | Customer |
| Customer Capacity | CC | Assign | Tactical | Customer |
| Multi-Depot | MD | Assign | Strategic | Depot |
| Time Windows for the Depot | WD | Evaluation | Strategic | Depot |
| Different end locations/Open Routes | O | Evaluation | Strategic | Depot |
| Different start and end locations | DA | Evaluation | Strategic | Depot |
| Departure from different locations | DD | Evaluation | Strategic | Depot |
| Precedence constraints | PC | Sequence | Tactical | Depot |
| Multi-Objectives | MO | Evaluation | Tactical | Depot |

## 7. INSIGHTS AND FUTURE TRENDS

From the previous sections, it is possible to extract some insight regarding the historical evolution of the VRP, both in terms of realism of the models they consider and the methods employed to solve them. Thus, as shown in Fig. 3, VRPs can be classified in three levels according to the degree of realism of associated models.

At the lower level, we find the most theoretical (classical) VRPs, which are represented by mostly academic models (as opposed to real-life models). These lab models are, of course, of high interest in order to develop mathematical and computing-based approaches mainly exact methods but also of heuristic nature. This way, solving techniques can be tested in controlled environments to assess their performance before being used in solving more complex models. The CVRP, VRPTW, HVRP and AVRP, among others, constitute clear examples of this category.

In a second level, the classical-advanced VRPs appear. These are models characterised by a higher level of realism: large-scale problems, multi-objective functions, combined routing and cross problems (e.g. VRPs combined with packing, allocation or inventory management), etc. More advanced and complex VRP variants are included in this category. Usually, these problems have been solved by metaheuristic approaches, such as GA, ACO, SA, GRASP, etc.
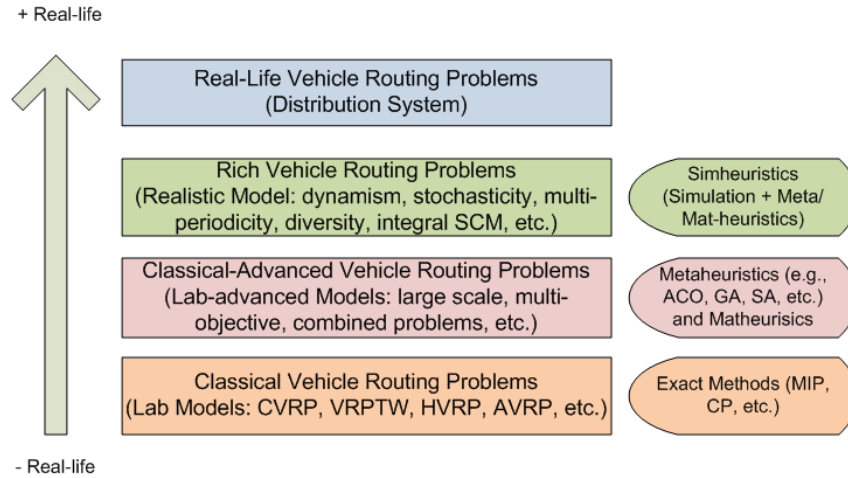
Fig. 3.   Models classification and methodological future trends in Rich Vehicle Routing Problems.

Most of the existing work on the VRP literature so far deals with the two aforementioned levels. Recently, however, and largely due to the matureness of existing exact and metaheuristic methods, researchers are able to go one step beyond and cope with Rich VRPs using a plethora of new hybrid methods, which combine exact and metaheuristic approaches (matheuristics) [Doerner and Schmid 2010] or even simulation with metaheuristics (simheuristics). As discussed in [Juan et al. 2014b], simheuristics allow considering uncertainty in costs and constraints of the VRP model, thus making these models to be a more accurate representation of real life routing distribution systems. These hybrid methods not only can deal with uncertainty (stochastic factors), but they can also consider aspects such as dynamism, diversity of vehicles and customers, multi-periodicity in the distribution activity, integration with other supply chain components, environmental issues, etc. As models and solving techniques are refined to tackle more realistic problems, a further increase of VRP variants considering complex constraints, and therefore included in the Rich VRP category, is to be expected.

## 8. CONCLUSIONS

The VRP is a classical combinatorial problem. Along history many different variants have been studied. The main difference among them is either the kind of constraints or the cost function of the specific type. Nowadays, it is common to find more and more complex problems, and closer to real world ones. These can be classified as Rich VRPs.

In this survey, we have reviewed the evolution of studied problems in the Rich VRP arena. We present a variety of routing scenarios that can be found in reality and the most common methods developed for addressing all types of Rich VRPs (i.e. exact, approximated, and their combinations). The Rich VRP domain has appeared on the first decade of the 21st century and it has shown itself as a promising research area. There are many tailored approaches for specific cases of Rich VRP. However, in the last ten years the general-purpose methods are slowly emerging keeping the previous quality features, but for generic Rich VRP scenarios.

In order to organise the information about the Rich VRP, we have analysed the different constraints included in Rich VRP papers, and tried to define how they can be characterised. Moreover, we have collected all the papers devoted to this area, classifying them according to the active constraints they have. Finally, we have included a

section that follows the evolution from the classical VRP to the so called Rich VRP, and makes an introduction of the future trends that this research line will face through the next years.

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